Integrability of an N-coupled nonlinear Schrödinger system for polarized optical waves in an isotropic medium via symbolic computation

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Considering the simultaneous propagation of multicomponent fields in an isotropic medium, an N-coupled nonlinear Schrödinger system with the self-phase modulation, cross-phase modulation, and energy exchange terms is investigated in this paper. First, via symbolic computation, the Painlevé singularity structure analysis shows that such a system admits the Painlevé property. Then, with the Ablowitz-Kaup-Newell-Segur scheme, the linear eigenvalue problem (Lax pair) associated with this model is constructed in the frame of the block matrices. With the Hirota bilinear method, the bright one- and two-soliton solutions of this system are presented. In addition, the bright multisoliton solutions of the system for $N=2$ are straightforwardly derived by the linear superposition of soliton solutions of two independent scalar nonlinear Schrödinger equations. Furthermore, through the analysis for the soliton solutions, the corresponding propagation behavior and applications for soliton pulses in nonlinear optical fibers are considered. Finally, three significant conserved quantities, i.e., energy, momentum, and Hamiltonian, are also given.

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I. INTRODUCTION

Since the theoretical prediction of optical solitons in the optical fibers by Hasegawa and Tappert in 1973 [1], along with the experimental demonstration of the transmission of envelope soliton in a single mode fiber [2], the propagation and application of optical solitons have attracted great interest both in theoretical and experimental studies over the last three decades [3–8]. Optical solitons are regarded as the natural data bits and as a promising alternative for the next generation of ultralong distance and extremely high bit rate optical telecommunication systems [3,4]. An optical soliton is known to arise when the group velocity dispersive and nonlinear effects of the fibers are well balanced. As an important physical model, the nonlinear Schrödinger (NLS) equation is used to describe the nonlinear pulse propagation in the optical fiber for a scalar field. Considering the simultaneous propagation of the soliton pulses for multiple fields with different frequencies or polarizations, the governing equation becomes the coupled nonlinear Schrödinger (CNLS) system in the study of soliton wavelength division multiplexing [9,10], multichannel bit parallel-wavelength optical fiber network [11], soliton switch in birefringent optical fibers [12–15], propagation and collision of the temporal vector solitons in birefringent fibers [16]. A CNLS system was first proposed [17] as

$$iq_{1_z} + q_{1_{tt}} + 2(|q_1|^2 + |q_2|^2)q_1 = 0,$$

$$iq_{2_z} + q_{2_{tt}} + 2(|q_2|^2 + |q_1|^2)q_2 = 0, \tag{1}$$

which governs the propagation of two fields with left and right polarizations in an optical fiber. This system possesses the Painlevé property [18] and its Lax pair has been constructed [19]. The soliton collisions with shape change by intensity redistribution have been discussed based on the bright multisoliton solutions obtained by using the Hirota bilinear method [20–22]. For the simultaneous propagation of N nonlinear waves, the corresponding governing equation is the N-CNLS system. The Painlevé property, Lax pair, and multisoliton solutions for the N-CNLS system have also been investigated in Refs. [18,19,22].

In an isotropic medium, the propagation of orthogonally polarized optical waves is governed by the following CNLS system [23–25]:

$$iq_{1_z} + q_{1_{tt}} + 2(|q_1|^2 + 2|q_2|^2)q_1 - 2q_1^* q_2^2 = 0,$$

$$iq_{2_z} + q_{2_{tt}} + 2(|q_2|^2 + 2|q_1|^2)q_2 - 2q_2^* q_1^2 = 0, \tag{2}$$

where q_1 and q_2 are slowly varying envelopes of two interacting optical modes, the variables z and t, respectively, correspond to the normalized distance and time, and the asterisk denotes the complex conjugate. Different from system (1), the ratio between the coefficients of the self-phase modulation and cross-phase modulation is 1:2. The last terms of system (2) represent the coherent coupling governing the energy exchange between two axes of the fiber [26]. The Painlevé analysis, Lax pair, and Hirota bilinear form have been presented in Ref. [23]. Reference [27] has obtained the multisoliton solutions and discussed the soliton interaction behaviors by virtue of the Darboux transformation.

In this paper, we devote our attention to the simultaneous propagation of N fields described by the following N-CNLS system:

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$$iq_{j_z}+q_{j_{tt}}+2\left(|q_j|^2+2\sum_{k=1}^{N}|q_k|^2\right)q_j-2\sum_{k=1}^{N}q_k^2q_j^*=0 \qquad (3)$$

$$(j,k=1,2,\ldots,N;\ k\neq j),$$

which is a generalization of system (2). For $N=4$, Ref. [24] has constructed the Lax pair and single soliton solution for the simultaneous propagation of four fields.

Symbolic computation is a new branch of artificial intelligence, which has been extensively used to deal with a large amount of complicated and tedious algebraic calculations in various fields of science and engineering [28]. With the help of the computerized symbolic computation, we will carry out the Painlevé singularity structure analysis to check the integrability of this system in Sec. II. In Sec. III, with the Ablowitz-Kaup-Newell-Segur (AKNS) scheme, we will derive the Lax pair in the frame of the block matrices. Section IV will be devoted to obtaining the bright one- and two-soliton solutions of system (3) by means of the Hirota bilinear method. In Sec. V, the multisoliton solutions of system (3) for $N=2$ will be constructed by the linear superposition of soliton solutions of two independent NLS equations. The corresponding propagation dynamics and applications for soliton pulses in nonlinear optical fibers will be considered. Three significant conserved quantities, i.e., energy, momentum and Hamiltonian, will be presented in Sec. VI. The last section will be our conclusions.

II. PAINLEVÉ ANALYSIS OF SYSTEM (3) WITH SYMBOLIC COMPUTATION

It is well known that the Painlevé singularity structure analysis has been a systematic and effective algorithmic method to identify the integrability of nonlinear evolution equations (NLEEs) in nonlinear science [29,30]. According to the Weiss-Tabor-Carnevale procedure [29], a given NLEE is said to have the Painlevé property when its solutions are single valued about the noncharacteristic movable singular manifold. In this section, based on this approach and the simplified Kruskal ansatz [30,31], we assume the solutions of system (3) in a generalized Laurent series expansions

$$q_j=\sum_{k=0}^{\infty}u_{jk}(z)\phi^{-\alpha_j+k}(z,t),$$

$$q_j^*=\sum_{k=0}^{\infty}v_{jk}(z)\phi^{-\beta_j+k}(z,t)\quad(j=1,2,\ldots,N), \qquad (4)$$

with $\phi(z,t)=t+\psi(z)$, where $\psi(z)$ is an arbitrary analytic function and $u_{jk}(z)$, $v_{jk}(z)$, $[u_{j0}(z),v_{j0}(z)\neq 0]$ are all analytical functions, in the neighborhood of a noncharacteristic movable singularity manifold defined by $\phi(z,t)=0$, while α_j and β_j are positive integers to be determined.

Determining the leading-order behavior, we introduce $q_j\approx u_{j0}\phi^{-\alpha_j}$ and $q_j^*\approx v_{j0}\phi^{-\beta_j}$ into system (3) and its complex conjugate version and obtain

$$\alpha_j=\beta_j=1,\quad u_{j0}=-v_{j0}\Bigg/\sum_{j=1}^{N}v_{j0}^2\quad(j=1,2,\ldots,N), \qquad (5)$$

from which we see that there are N arbitrary functions v_{j0} $(j=1,2,\ldots,N)$. For finding the resonances (namely, the powers at which arbitrary functions can enter the Laurent series), we now substitute

$$q_j=u_{j0}\phi^{-1}+u_{jk}\phi^{-1+k},\quad q_j^*=v_{j0}\phi^{-1}+v_{jk}\phi^{-1+k}, \qquad (6)$$

into system (3) and its complex conjugate version, and equate the coefficients of $(\phi^{-3},\ldots,\phi^{-3})$ to get

$$\begin{vmatrix}
A & \frac{2(2v_{10}^2-B)}{B^2} & 0 & \frac{4v_{10}v_{20}}{B^2} & 0 & \frac{4v_{10}v_{30}}{B^2} & \cdots & 0 & \frac{4v_{10}v_{N0}}{B^2}\\
\frac{2(2v_{10}^2-B)}{1} & A & 4v_{10}v_{20} & 0 & 4v_{10}v_{30} & 0 & \cdots & 4v_{10}v_{N0} & 0\\
0 & \frac{4v_{10}v_{20}}{B^2} & A & \frac{2(2v_{20}^2-B)}{B^2} & 0 & \frac{4v_{20}v_{30}}{B^2} & \cdots & 0 & \frac{4v_{20}v_{N0}}{B^2}\\
4v_{10}v_{20} & 0 & \frac{2(2v_{20}^2-B)}{1} & A & 4v_{20}v_{30} & 0 & \cdots & 4v_{20}v_{N0} & 0\\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots\\
0 & \frac{4v_{10}v_{N0}}{B^2} & 0 & \frac{4v_{20}v_{N0}}{B^2} & 0 & \frac{4v_{30}v_{N0}}{B^2} & \cdots & A & \frac{2(2v_{N0}^2-B)}{B^2}\\
4v_{10}v_{N0} & 0 & 4v_{20}v_{N0} & 0 & 4v_{30}v_{N0} & 0 & \cdots & \frac{2(2v_{N0}^2-B)}{1} & A
\end{vmatrix}=0, \qquad (7)$$

with

$$A = (k-2)(k-1)\phi_t^2 - 4, \quad B = \sum_{j=1}^{N} v_{j0}^2.$$

By setting the determinant to zero, the resonances are given as

$$k = \overbrace{-1, \ldots, -1}^{N}, \quad \overbrace{0, \ldots, 0}^{N}, \quad \overbrace{3, \ldots, 3}^{N}, \quad \overbrace{4, \ldots, 4}^{N}. \quad (8)$$

The resonance at $k=-1$ naturally represents the arbitrariness of the singular manifold $\phi(z,t)$, while

$$k = \overbrace{0, \ldots, 0}^{N}$$

are associated with the arbitrariness of the functions v_{j0} $(j=1,2,\ldots,N)$ as seen in expression (5). With the aid of symbolic computation, the existence of sufficient number of arbitrary functions at the resonance values

$$k = \overbrace{3, \ldots, 3}^{N} \quad \text{and} \quad k = \overbrace{4, \ldots, 4}^{N}$$

can be easily checked by substituting the full Laurent expansions (4) into system (3) and analyzing the equations by collecting the coefficients of different powers of ϕ. Thus far, it has been shown that the solutions of system (3) admit the required number of arbitrary functions without the introduction of any movable critical manifold. Therefore, we can conclude that system (3) possesses the Painlevé property. It is interesting to note that all resonance $k=-1,0,3,4$ are all the N multiple roots, but the simple roots $k=-1,0,3,4$ are precisely those of the standard NLS equation. In Ref. [23], by making use of the Painlevé singularity structure analysis, the authors have found that all resonances of system (2) are the double poles, i.e., $k=-1,-1,0,0,3,3,4,4$, which means that 2-CNLS can be decoupled into two independent NLS equations, and is also in conformity with the existence of double -1 values in the resonances.

III. LAX PAIR OF SYSTEM (3)

An important feature of many integrable NLEEs is that they can be thought of as the compatibility conditions for the linear eigenvalue equations (Lax pair) having a spectral parameter [30]. The Lax pair plays an important role in obtaining the solutions, integrable properties and the initial value problem for a given integrable NLEE solved by use of the inverse scattering transform method [30]. The AKNS scheme provides a very systematic way to construct the Lax pair, whose compatibility conditions yield many completely integrable models of physical interest [32–34]. By means of such a scheme, the Lax pair of system (3) can be constructed with the following linear eigenvalue problem:

$$\Psi_t = \mathbf{U}\Psi = (\lambda U_0 + U_1)\Psi, \quad \Psi_z = \mathbf{V}\Psi = (\lambda^2 V_0 + \lambda V_1 + V_2)\Psi, \quad (9)$$

with the block matrices U_0, U_1, V_0, V_1, V_2 as

$$U_0 = i\begin{pmatrix} -\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{pmatrix}, \quad U_1 = \begin{pmatrix} \mathbf{0} & \mathbf{Q} \\ -\mathbf{Q}^\dagger & \mathbf{0} \end{pmatrix}, \quad (10)$$

$$V_0 = 2i\begin{pmatrix} -\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{pmatrix}, \quad V_1 = 2\begin{pmatrix} \mathbf{0} & \mathbf{Q} \\ -\mathbf{Q}^\dagger & \mathbf{0} \end{pmatrix},$$

$$V_2 = i\begin{pmatrix} \mathbf{Q}\cdot\mathbf{Q}^\dagger & \mathbf{Q}_t \\ \mathbf{Q}_t^\dagger & -\mathbf{Q}^\dagger\cdot\mathbf{Q} \end{pmatrix}, \quad (11)$$

where Ψ is a $2N$-dimensional vector function, $\mathbf{I}$ is an unit matrix, λ is the spectral parameter independent of z and t, $\mathbf{Q}$ is a square matrix, and the sword denotes the Hermitian conjugate. The compatibility condition for Eq. (9), i.e., the zero-curvature equation $\mathbf{U}_z - \mathbf{V}_t + [\mathbf{U},\mathbf{V}] = 0$, where the brackets denote a commutator of two matrices, yields the following matrix NLS equation:

$$i\mathbf{Q}_z + \mathbf{Q}_{tt} + 2\mathbf{Q}\cdot\mathbf{Q}^\dagger\cdot\mathbf{Q} = 0, \quad (12)$$

from which families of CNLS systems are able to be derived according to different forms of $\mathbf{Q}$. For the special case $\mathbf{Q} = \mathbf{Q}_1 = q$, Eq. (12) leads to the standard NLS equation

$$iq_z + q_{tt} + 2|q|^2 q = 0. \quad (13)$$

If $\mathbf{Q}$ is chosen as the following forms:

$$\mathbf{Q}_2 = \left(\begin{array}{c|c} q_1 & q_2 \\ \hline -q_2 & q_1 \end{array}\right), \quad (14)$$

$$\mathbf{Q}_3 = \left(\begin{array}{cc|cc} q_1 & q_2 & q_3 & 0 \\ -q_2 & q_1 & 0 & q_3 \\ \hline -q_3 & 0 & q_1 & -q_2 \\ 0 & -q_3 & q_2 & q_1 \end{array}\right), \quad (15)$$

$$\mathbf{Q}_4 = \left(\begin{array}{cccc|cccc} q_1 & q_2 & 0 & 0 & q_3 & 0 & q_4 & 0 \\ -q_2 & q_1 & 0 & 0 & 0 & q_3 & 0 & q_4 \\ 0 & 0 & q_1 & q_2 & -q_4 & 0 & q_3 & 0 \\ 0 & 0 & -q_2 & q_1 & 0 & -q_4 & 0 & q_3 \\ \hline -q_3 & 0 & q_4 & 0 & q_1 & -q_2 & 0 & 0 \\ 0 & -q_3 & 0 & q_4 & q_2 & q_1 & 0 & 0 \\ -q_4 & 0 & -q_3 & 0 & 0 & 0 & q_1 & -q_2 \\ 0 & -q_4 & 0 & -q_3 & 0 & 0 & q_2 & q_1 \end{array}\right), \tag{16}$$

the 2-CNLS, 3-CNLS, and 4-CNLS systems are obtained by substituting expressions (14)–(16) into Eq. (12), respectively. In what follows, we give the general expression of $\mathbf{Q}_N$ as a $2^{N-1}\times 2^{N-1}$ block matrix

$$\mathbf{Q}_N = \left(\begin{array}{c|c} \mathbb{Q}_1 & \mathbb{Q}_2 \\ \hline \mathbb{Q}_3 & \mathbb{Q}_4 \end{array}\right), \tag{17}$$

where $\mathbb{Q}_j$ $(j=1,2,3,4)$ are all $2^{N-2}\times 2^{N-2}$ square-block matrices, $\mathbb{Q}_1$ is a block diagonal matrix, while $\mathbb{Q}_3=-\mathbb{Q}_2^T$, $\mathbb{Q}_4=\mathbb{Q}_1^T$ (T denotes the transpose of the matrix). $\mathbb{Q}_1$ and $\mathbb{Q}_2$ are given by

$$\mathbb{Q}_1 = \begin{pmatrix} \mathbb{A}_1 & & & & \\ & \mathbb{A}_1 & & 0 & \\ & & \ddots & & \\ & 0 & & \mathbb{A}_1 & \\ & & & & \mathbb{A}_1 \end{pmatrix}, \quad \mathbb{Q}_2 = \left(\begin{array}{c|c} \mathbb{B}_1 & \mathbb{B}_2 \\ \hline \mathbb{B}_3 & \mathbb{B}_4 \end{array}\right), \tag{18}$$

where $\mathbb{Q}_2$ has the same identities as $\mathbf{Q}_N$, i.e., $\mathbb{B}_j$ are all square-block matrices, $\mathbb{B}_3=-\mathbb{B}_2^T$, $\mathbb{B}_4=\mathbb{B}_1^T$, while $\mathbb{B}_1$ and $\mathbb{B}_2$ are expressible in the form

$$\mathbb{B}_1 = \begin{pmatrix} \mathbb{A}_2 & & & \\ & \mathbb{A}_2 & & 0 \\ & & \ddots & \\ & 0 & & \mathbb{A}_2 \end{pmatrix},$$

$$\mathbb{B}_2 = \begin{pmatrix} \mathbb{A}_3 & \mathbb{O} & \cdots & \mathbb{A}_{N-1} & \mathbb{A}_N \\ \mathbb{O} & \mathbb{A}_3 & \cdots & -\mathbb{A}_N & \mathbb{A}_{N-1} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -\mathbb{A}_{N-1} & \mathbb{A}_N & \cdots & \mathbb{A}_3 & \mathbb{O} \\ -\mathbb{A}_N & -\mathbb{A}_{N-1} & \cdots & \mathbb{O} & \mathbb{A}_3 \end{pmatrix}, \tag{19}$$

$$\mathbb{A}_1 = \begin{pmatrix} q_1 & q_2 \\ -q_2 & q_1 \end{pmatrix}, \quad \mathbb{O} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix},$$

$$\mathbb{A}_j = \begin{pmatrix} q_{j+1} & 0 \\ 0 & q_{j+1} \end{pmatrix} \quad (j=2,3,\ldots,N). \tag{20}$$

It is easy to verify that system (3) can be derived from Eq. (12) with the substitution of expressions (17)–(20).

IV. SOLITON SOLUTIONS OF SYSTEM (3) WITH THE HIROTA BILINEAR METHOD

In this section, using the Hirota bilinear method, we will construct the exact bright one- and two-soliton solutions of system (3).

By the bilinear transformation $q_j=g^{(j)}/f$, system (3) can be transformed into the bilinear form

$$(D_z - iD_t^2)(g^{(j)}\cdot f) = 0, \tag{21}$$

$$D_t^2(f\cdot f) = 2\left(|g^{(j)}|^2 + 2\sum_{k=1}^{N}|g^{(k)}|^2\right) - 2\frac{g^{(j)*}}{g^{(j)}}\sum_{k=1}^{N} g^{(k)^2}$$

$$(j,k=1,2,\ldots,N;\ k\neq j), \tag{22}$$

where D is the well-known Hirota bilinear operator [35]

$$D_z^m D_t^n g\cdot f = (\partial_z - \partial_{z'})^m(\partial_t - \partial_{t'})^n g(z,t)f(z',t')\big|_{z'=z,t'=t}. \tag{23}$$

The soliton solutions of system (3) can be constructed by expanding the functions $g^{(j)}$ and f as power series of small parameter ε

$$g^{(j)} = \varepsilon g_1^{(j)} + \varepsilon^3 g_3^{(j)} + \varepsilon^5 g_5^{(j)} + \cdots,$$

$$f = 1 + \varepsilon^2 f_2 + \varepsilon^4 f_4 + \cdots \quad (j=1,2,\ldots,N). \tag{24}$$

A. Bright one-soliton solution

Through the terminated expansion $g^{(j)}=\varepsilon g_1^{(j)}$, $f=1+\varepsilon^2 f_2$, the bright one-soliton solution can be obtained with solving Eqs. (21) and (22)

$$q_j = \frac{\alpha_1^{(j)} e^{\zeta_1}}{1+e^{\zeta_1+\zeta_1^*+\sigma}} = \frac{\alpha_1^{(j)}}{2} e^{-(\sigma/2)}\,\mathrm{sech}\left[\mathrm{Re}(\zeta_1) + \frac{\sigma}{2}\right] e^{i\,\mathrm{Im}(\zeta_1)}, \tag{25}$$

where $e^{\sigma}=\sum_{j=1}^{N}|\alpha_1^{(j)}|^2/(k_1+k_1^*)$, $\zeta_1=k_1t+ik_1^2z$, k_1 is an arbitrary complex constant, and $\alpha_1^{(j)}$ are all arbitrary real constants. Fig. 1 shows the intensity plots of bright one-soliton solution for the 3–CNLS equations.

B. Bright two-soliton solution

Expanding $g^{(j)}=\varepsilon g_1^{(j)}+\varepsilon^3 g_3^{(j)}$, $f=1+\varepsilon^2 f_2+\varepsilon^4 f_4$, we can get the bright two-soliton solution from Eqs. (21) and (22)

$$q_j=\frac{\alpha_1^{(j)}e^{\zeta_1}+\alpha_2^{(j)}e^{\zeta_2}+e^{\zeta_1+\zeta_1^*+\zeta_2+\gamma_{1j}}+e^{\zeta_1+\zeta_2+\zeta_2^*+\gamma_{2j}}}{1+e^{\zeta_1+\zeta_1^*+\tau_1}+e^{\zeta_1+\zeta_2^*+\tau_0}+e^{\zeta_1^*+\zeta_2+\tau_0^*}+e^{\zeta_2+\zeta_2^*+\tau_2}+e^{\zeta_1+\zeta_1^*+\zeta_2+\zeta_2^*+\tau_3}},\tag{26}$$

with

$$e^{\gamma_{1j}}=\frac{\kappa^2\alpha_2^{(j)}(k_1-k_2)^2\omega}{(k_1+k_1^*)^2(k_2+k_1^*)^2},\quad e^{\gamma_{2j}}=\frac{\kappa\alpha_2^{(j)}(k_1-k_2)^2\omega}{(k_1+k_2^*)^2(k_2+k_2^*)^2},$$

$$e^{\tau_0}=\frac{\kappa\omega}{(k_2^*+k_1)^2},\tag{27}$$

$$e^{\tau_1}=\frac{\kappa^2\omega}{(k_1+k_1^*)^2},\quad e^{\tau_2}=\frac{\omega}{(k_2+k_2^*)^2},$$

$$e^{\tau_3}=\frac{\kappa^2|k_1-k_2|^4\omega^2}{(k_1+k_1^*)^2|k_2+k_1^*|^4(k_2+k_2^*)^2},\tag{28}$$

$$\zeta_1=k_1t+ik_1^2z,\quad \zeta_2=k_2t+ik_2^2z,$$

$$\omega=\sum_{m=1}^{N}|\alpha_2^{(m)}|^2,\quad \alpha_1^{(j)}=\kappa\alpha_2^{(j)},\tag{29}$$

where k_1 and k_2 are two arbitrary complex constants, while κ and $\alpha_2^{(j)}$ are all arbitrary real constants. Fig. 2 displays the elastic collision of two bright soliton for the 3–CNLS equations.2).

V. OPTICAL SOLITON SOLUTIONS AND ITS APPLICATIONS OF SYSTEM (3) FOR $N=2$

In the above section, the soliton collisions taking place in respective component are all elastic because of the following relationship:

$$\frac{\alpha_1^{(1)}}{\alpha_2^{(1)}}=\frac{\alpha_1^{(2)}}{\alpha_2^{(2)}}=\cdots=\frac{\alpha_1^{(N)}}{\alpha_2^{(N)}}=\kappa.\tag{30}$$

In this section, we will turn our attention to the 2-CNLS equations of system (3). In Ref. [23], it has been noted that through substituting $\Psi_1=q_1+iq_2$ and $\Psi_2=q_1-iq_2$ into the two independent NLS equations

$$i\Psi_{j_z}+\Psi_{j_{tt}}+2|\Psi_j|^2\Psi_j=0\quad (j=1,2),\tag{31}$$

system (2) can be recovered, which means that system (2) can be decoupled into two independent NLS equations, and the solution of system (2) is available from the linear superposition of the solutions of these two independent NLS equaitons. In this section, with such a valid superposition principle, i.e., $q_1=\frac{1}{2}(\Psi_1+\Psi_2)$ and $q_2=-i\frac{1}{2}(\Psi_1-\Psi_2)$, we will obtain the bright soliton solutions of system (2) by the bright soliton solutions of Eq. (31).

Substituting the dependent variable transformations $\Psi_j=G_j/F_j$ $(j=1,2)$ into Eq. (31), we can get the bilinear forms [35]

$$(iD_z+D_t^2)(G_j\cdot F_j)=0,\quad D_t^2(F_j\cdot F_j)=2G_jG_j^*\quad (j=1,2).\tag{32}$$

After symbolic computation on Eq. (32), the one- and two-soliton solutions of Eq. (31) are presented as follows.

1. One-soliton solution

$$\Psi_j^{(1)}=\frac{\alpha_j}{2}e^{-(\rho_j/2)}\operatorname{sech}\{k_{jR}[t-\xi_j(z)]\}e^{i\varphi_j(z,t)}\quad (j=1,2),\tag{33}$$

where $e^{\rho_j}=\frac{|\alpha_j|^2}{(k_j+k_j^*)^2}$, $k_j=k_{jR}+k_{jI}i$, $\xi_j(z)=k_{jI}z-\rho_j/2k_{jR}$, $\varphi_j(z,t)=k_{jI}t+(k_{jR}^2-k_{jI}^2)z$, and α_j are arbitrary complex parameters, while the subscripts R and I on k_j represent the real and imaginary parts.

2. Two-soliton solution

$$\Psi_j^{(2)}=\frac{\alpha_je^{\theta_j}+\beta_je^{\eta_j}+e^{\theta_j+\theta_j^*+\eta_j+\gamma_j}+e^{\theta_j+\eta_j+\eta_j^*+\delta_j}}{1+e^{\theta_j+\theta_j^*+\rho_j}+e^{\theta_j+\eta_j^*+\vartheta_j}+e^{\theta_j^*+\eta_j+\vartheta_j^*}+e^{\eta_j+\eta_j^*+\nu_j}+c_je^{\theta_j+\theta_j^*+\eta_j+\eta_j^*}}\quad (j=1,2),\tag{34}$$

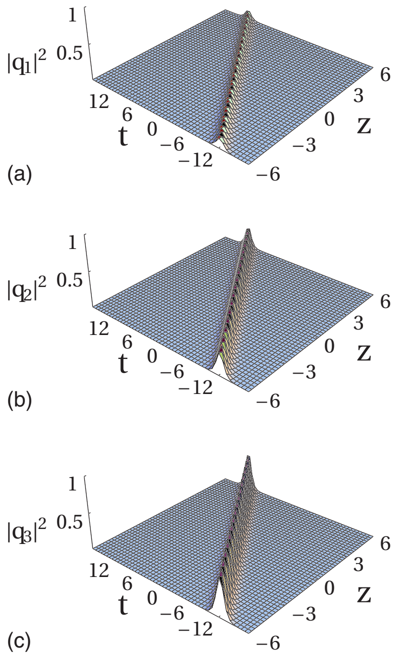

FIG. 1. (Color online) Intensity plots of one-soliton solution (25) for the case $N=3$. The relevant parameters are chosen as $k_1=1+i$, $\alpha_1^{(1)}=1$, $\alpha_1^{(2)}=1.5$, and $\alpha_1^{(3)}=2$.

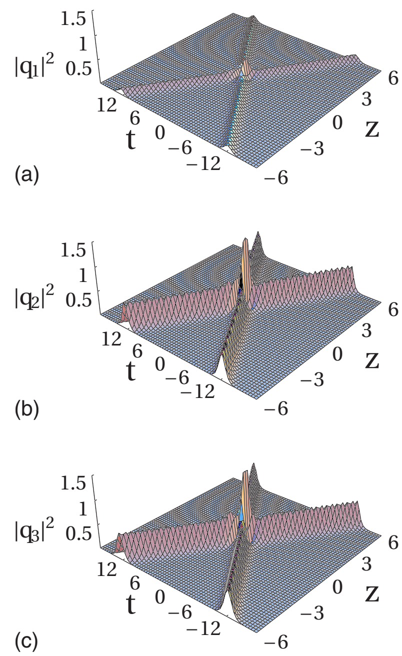

FIG. 2. (Color online) Elastic collision of two solitons expressed by solution (26) for the case $N=3$. The relevant parameters are chosen as $k_1=1+i$, $k_2=1-i$, $\alpha_1^{(1)}=\alpha_2^{(1)}=1$, $\alpha_1^{(2)}=\alpha_2^{(2)}=2$, and $\alpha_1^{(3)}=\alpha_2^{(3)}=1.5$.

where

$$\theta_j = k_j t + i k_j^2 z,$$

$$\eta_j = l_j t + i l_j^2 z,$$

$$c_j = \frac{|\alpha_j|^2|\beta_j|^2(k_j-l_j)^2(k_j^*-l_j^*)^2}{(k_j+k_j^*)^2(l_j+l_j^*)^2(k_j+l_j^*)^2(l_j+k_j^*)^2},$$

$$e^{\vartheta_j} = \frac{\alpha_j\beta_j^*}{(k_j+l_j^*)^2},$$

$$e^{\gamma_j} = \frac{\beta_j|\alpha_j|^2(k_j-l_j)^2}{(k_j+k_j^*)^2(l_j+k_j^*)^2},$$

$$e^{\delta_j} = \frac{\alpha_j|\beta_j|^2(k_j-l_j)^2}{(l_j+l_j^*)^2(k_j+l_j^*)^2},$$

$$e^{\nu_j} = \frac{|\beta_j|^2}{(l_j+l_j^*)^2},$$

while α_j, β_j, k_j, l_j are all arbitrary complex parameters.

Thus, the bright two-soliton solutions of system (2) can be straightforwardly gained from the linear superposition of two single solitons

$$q_1 = \frac{1}{2}(\Psi_1^{(1)} + \Psi_2^{(1)}), \quad q_2 = -\frac{1}{2}i(\Psi_1^{(1)} - \Psi_2^{(1)}). \qquad (35)$$

The existence of such a superposition principle also provides many useful insight into the structures of soliton solutions and interaction between solitons. The effectiveness of this approach has been demonstrated in Refs. [36–38], where the behavior and interaction between solitons have been considered with the two-soliton solution as a sum of two single solitons. It can be seen from expression (33) and solution (35) that $\text{sech}\{k_{jR}[t-\xi_j(z)]\}$ represents the shape of the bright soliton, the soliton amplitude is $\frac{1}{2}|k_{jR}|$, the width of the soliton is $\frac{1}{|k_{jR}|}$, $\varphi_j(z,t)$ is the soliton phase, and $\xi_j(z)$ is the soliton position. The distance between two solitons is $\Delta d=|\xi_1(z)-\xi_2(z)|$ which is an important parameter to suppress the overlap phenomenon between two solitons. Figure 3 shows the stable propagation of a single soliton pulse in each component, when solution (35) become a single soliton. In the optical communication system, in order to increase the information-carrying capacity of the fiber, the multisoliton pulses are often launched. However, the mutual interaction will will bring about the overlap between neighboring soli-

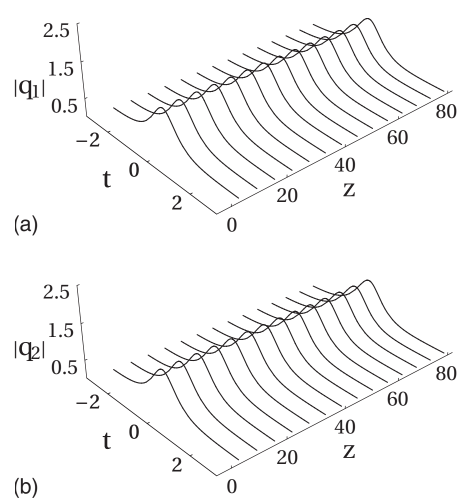

FIG. 3. Stable propagation of bright soliton pulse via solutions (35) with $k_1=-2$, $\alpha_1=2$, $k_2=1$, and $\alpha_2=0$.

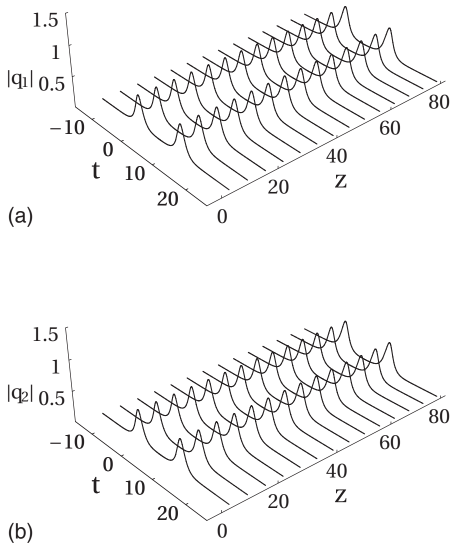

FIG. 4. Stable propagation of two parallel bright soliton pulses via solutions (35) with $k_1=-1$, $\alpha_1=e\times10^{-1}$, $k_2=1$, $\alpha_2=e\times10^{-5}$, and $\Delta d=13.2018$.

tons. Many efforts have been devoted to controlling the mutual interaction so as to improve the transmission rate. Figure 4 displays that two soliton pulses with equal amplitude propagate independently retaining the invariant separation distance $\Delta d=13.2018$. In comparison, Fig. 5 provides two cases of the overlap of two soliton pulses with the same amplitude or different amplitudes. The above results show that the overlap between two soliton pulses can be controlled by adjusting the parameter Δd appropriately.

In optical communication systems, the all-optical switching [39], construction of logic gates [21,22,40], information transformation [41], as well as optical computing [3,16,25]

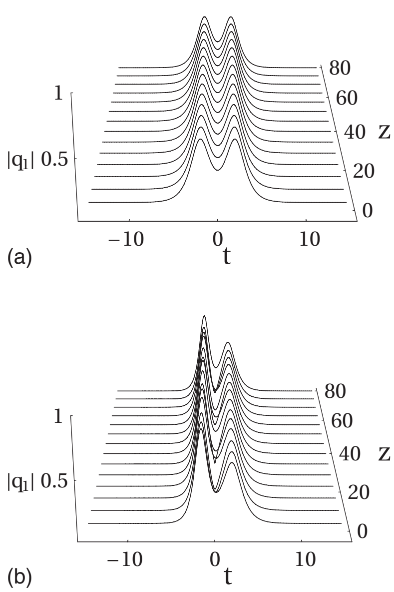

FIG. 5. (a) Soliton overlap with the equal amplitude. The parameters are the same as those in Fig. 4 except that $\alpha_2=e\times10^{-1}$ and $\Delta d=3.9915$. (b) Soliton overlap with different amplitudes. The parameters are the same as those in Fig. 4 except that $k_1=-1.5$, $\alpha_2=e\times10^{-1}$, and $\Delta d=3.5965$.

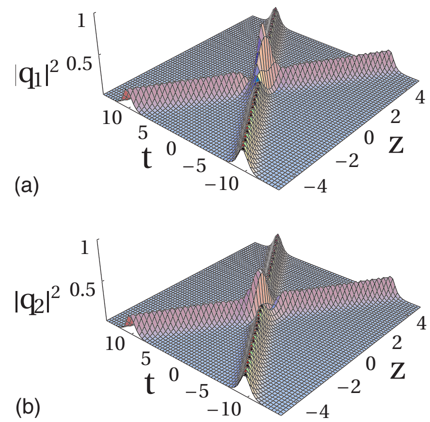

FIG. 6. (Color online) Intensity plots of bright soliton solutions (35) by the linear superposition of two single soliton solutions (33) with $k_1=1+i$, $\alpha_1=1+i$, $k_2=-1-i$, and $\alpha_2=2$.

are based on the context of the interaction of vector solitons. Due to the multicomponent structure of CNLS system, the vector solitons can exhibit richer propagation dynamics and collision properties than one-component counterparts. Therefore, it is of great interest in studies for the interaction of vector solitons both in theory and experiment [8,42–47]. It has been shown that the bright vector solitons undergo the fascinating intensity shape-changing collisions due to intensity redistributions in two components [20,22]. In the collision process, the intensity of the bright vector solitons can be enhanced or suppressed, and even completely annihilated. In addition, it has been experimentally demonstrated that the energy-exchange interaction of vector solitons is possible [49,48,50], and how information can pass from one collision to the next when the optical pulses act as the information carries in two component fields [41]. In the following, Figs. 6–9 display the intensity profiles of bright vector soliton solutions obtained by the linear superposition of the single soliton and two-soliton solutions. In Figs. 6 and 7, the intensities of bright solitons are invariant in the propagation process. However, in Figs. 8 and 9, if the intensity of soliton gets

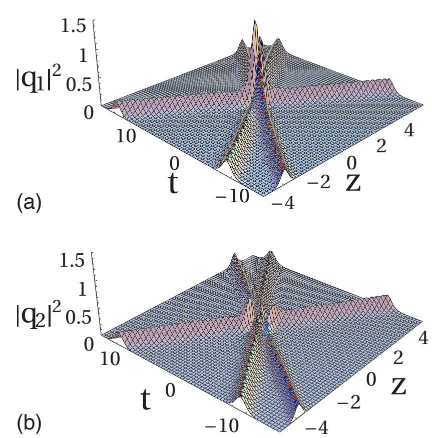

FIG. 7. (Color online) Intensity plots of bright soliton solution by the linear superposition of a single soliton solution (33) and a two-soliton solution (34) with $k_1=1+2i$, $\alpha_1=1$, $l_1=1-i$, $\beta_1=1$, $k_2=1+i$, and $\alpha_2=1$.

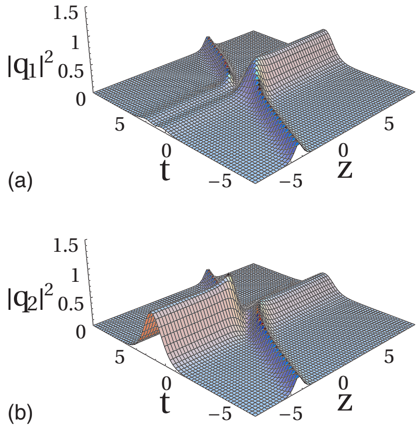

FIG. 8. (Color online) Intensity plots of bright soliton solution by the linear superposition of a single soliton solution (33) and a two-soliton solution (34) with $k_1=-1$, $\alpha_1=1+4i$, $l_1=1+i$, $\beta_1=1$, $k_2=1$, and $\alpha_2=2$.

enhanced (suppressed) in one component, while in the other component the corresponding soliton intensity is suppressed (enhanced). In the particular case, the intensity can be completely annihilated.

VI. CONSERVED QUANTITIES OF SYSTEMS (3)

In Figs. 8 and 9, although the intensity redistribution for the solitons between two component fields, the total energy of the system is conserved. In this section, the total energy, momentum and Hamiltonian of system (3) are presented. These quantities are useful for analytical investigation of the stability for the soliton pulses in optical communications [51,52].

Multiplying the jth equation of system (3) and its complex conjugate system by q_j^* and q_j, respectively, and implementing the subtraction operation for those resultant equations yield the energy conservation law for the entire bright vector solitons of systems (3)

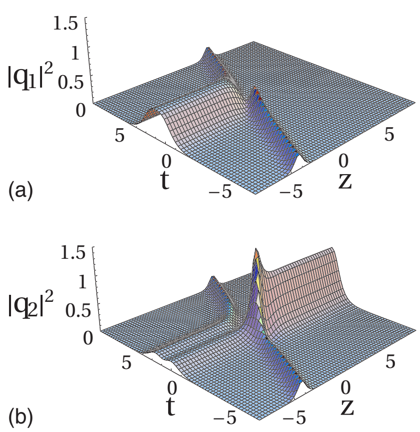

FIG. 9. (Color online) Intensity plots of bright soliton solution by the linear superposition of a single soliton solution (33) and a two-soliton solution (34) with $k_1=-1$, $\alpha_1=8i$, $l_1=1+i$, $\beta_1=1$, $k_2=1$, and $\alpha_2=2i$.

$$i\frac{\partial}{\partial z}\left(\sum_{j=1}^{N}|q_j|^2\right)=\frac{\partial}{\partial t}\left(\sum_{j=1}^{N}q_j q_{j_t}^*-\sum_{j=1}^{N}q_j^* q_{j_t}\right). \tag{36}$$

By virtue of the vanishing boundary conditions $q_j|_{t\to\pm\infty}\to 0$, the total energy of system (3) is given by

$$\mathcal{P}=\int_{-\infty}^{+\infty}\sum_{j=1}^{N}|q_j|^2 dt. \tag{37}$$

Proceeding further as above, we can write other two conserved equations as

$$\begin{aligned}&i\frac{\partial}{\partial z}\left[\sum_{j=1}^{N}(q_j^* q_{j_t}-q_j q_{j_t}^*)\right]\\&\quad=\frac{\partial}{\partial t}\left[2\sum_{j=1}^{N}(|q_{j_t}|^2-|q_j|^4)-\sum_{j=1}^{N}(q_{j_{tt}}q_j^*+q_j q_{j_{tt}}^*)\right.\\&\qquad\left.-8\sum_{1\leqslant k<m\leqslant N}|q_k|^2|q_m|^2+2\sum_{j=1}^{N}q_j^2\sum_{k=1}^{N(k\neq j)}q_k^{*2}\right],\end{aligned} \tag{38}$$

$$\begin{aligned}&\frac{\partial}{\partial t}\left[\sum_{j=1}^{N}(q_{j_t}q_{j_z}^*+q_{j_t}^* q_{j_z})\right]\\&\quad=\frac{\partial}{\partial z}\left[\sum_{j=1}^{N}(|q_{j_t}|^2-|q_j|^4)-4\sum_{1\leqslant k<m\leqslant N}|q_k|^2|q_m|^2\right.\\&\qquad\left.+\sum_{j=1}^{N}q_j^2\sum_{k=1}^{N(k\neq j)}q_k^{*2}\right],\end{aligned} \tag{39}$$

from which the total momentum and Hamiltonian of system (3) are, respectively, given by

$$\mathcal{M}=i\int_{-\infty}^{+\infty}\sum_{j=1}^{N}(q_j^* q_{j_t}-q_j q_{j_t}^*)dt, \tag{40}$$

$$\begin{aligned}\mathcal{H}=\int_{-\infty}^{+\infty}&\left[\sum_{j=1}^{N}(|q_{j_t}|^2-|q_j|^4)-4\sum_{1\leqslant k<m\leqslant N}|q_k|^2|q_m|^2\right.\\&\left.+\sum_{j=1}^{N}q_j^2\sum_{k=1}^{N(k\neq j)}q_k^{*2}\right]dt.\end{aligned} \tag{41}$$

VII. CONCLUSIONS

In this paper, we have investigated an N-CNLS system with the self-phase modulation, cross-phase modulation, and energy exchange terms, which can be used to describe the simultaneous propagation of N fields in an isotropic medium. Performing the Painlevé singularity structure analysis, we have identified that such a system possesses the Painlevé property. Based on the AKNS scheme, the linear eigenvalue problem associated with this N-CNLS system has also been constructed in the frame of the block matrices. Furthermore, we have constructed the bright one- and two-soliton solutions of this system by employing the Hirota method. When $N=2$, it has been shown that the multisoliton solutions can be straightforwardly available by the linear superposition of the soliton solutions of two independent scalar NLS equations. According to the obtained soliton solutions and graphical analysis, we have discussed the following applications of the soliton pulses propagation in nonlinear optical fibers: (1) the overlap phenomenon between two closely spaced soliton pulses and (2) intensity preservation and redistribution between two components. Finally, three significant conserved quantities have been given.

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